Highway Traffic Flow Models under Specific Conditions

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Abstract
This study aims to accurately estimate and predict passenger car speed under special conditions. Two special conditions were investigated in this research study; highway quasi-free-flow condition and highway speed control condition. Using statistical analysis the compatibility rate of three famous models on the data, are measured. Then for higher model compatibility and better mean speed prediction, two new models were proposed. Results showed poor compatibility of the three famous models in highway quasi-free-flow condition and partial compatibility in highway stream with passenger car speed limitation. In highway speed control condition, using the proposed model, the mean speed could be predicted with higher accuracy than the other three models. Also, Using developed models by new parameters, passenger car mean speed could be predicted more precision in highway quasi-free-flow condition.

Keywords: ***Quasi-Free-Flow, speed, density, flow***

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Introduction

Since 1935 when Greenshields presented a relationship between traffic speed and density, there have been several research efforts on this topic [1]. Greenshileds model was a simple model for training but its accordance was always anguishing. This model was defined by a linear relationship between free flow speed, jam density and vehicle density [2].

In 1955 Lighthill and Whitham proposed flow rate as a product of speed and flow [3]. Then, In 1959 Greenberg introduced speed as a logarithmic function of optimum speed, jam density and vehicle density [4]. Greenberg's model was considered as a bridge between macroscopic flow model and microscopic car-following model [3]. In 1961 Underwood proposed speed as an exponential function of free flow speed, optimum speed and maximum density. In This model speed never reaches zero and jam density is infinity [5]. Pipes and Louis, In 1967, developed Greenshields model by adding the powers "n" and "m" to it and calculated the values of "n" and "m" at different conditions [6]. Another research was conducted by Kerner and Konhauser In 1994 .They used a new relationship between speed and density in Homogenous traffic stream [7].

Also, Daganzo proposed a method to predict traffic's evolution over time and space. The proposed method automatically generates suitable variations in density at locations where the hydrodynamic theory would call for a shockwave; i.e., a jump in density [8]. In free flows, Wong et.al proposed the uniform relationship between speed and density with different speed. In this effort, they classified vehicles in 9 classes by their free flow speeds from 60km//h to 120km/h with 7.5 km/h intervals [9].

In 2008, Nielsen et.al concluded that a linearly decreasing density- flow relationship may be assumed for the queuing situation, although less clearly than for the non-queuing situation [10]. In this year, another research was performed by Xiaowen et.al. They proposed a technique for estimation of traffic density on the basis of traffic flow and time mean speed on urban freeway [11].In 2011, Rui et.al analyzed Greeenshields, Pipes and Louis, Underwood, improved Underwood model using data collected in urban expressway. They found that improved Underwood model is mostly suitable with praiseworthy Fitting result, especially for traffic condition of urban expressway [12]. Also, the relationship between speed and flow rate using the data acquired with radar traffic counter system in a freeway section

.This study was carried out with consideration of different climate conditions, different surface conditions(i.e. dry, wet and icy) and percentage of heavy vehicle [13].Three new models for the flow–density relationship were proposed by Dell Castillo. The test performed using data obtained from freeway and urban showed an excellent goodness of fit. Four parameters was used in these models that include the jam density, the free-flow speed, the kinematic wave speed and a shape parameter [14].
Van Wageningen-Kessels et.al analyze hyperbolicity and anisotropy of multi-class models. They derived a generic formulation of multi-class kinematic wave traffic flow models. Analysis showed that most multi-class kinematic wave traffic flow models are indeed hyperbolic and anisotropic under certain modeling conditions [15].
In this paper, the fitting effectiveness of the three models, Greenshields, Greenberg and Underwood is determined. Then, for better compatibility with empirical data, a new model is proposed and finally the model evaluation is discussed.

Problem Definition

Considering that driver behavior varies in different regions of the world, examining the adaption rate of some known models is necessary in other regions [16]. Therefore, this paper studies the adaption rate of *Greenshields*, *Greenberg* and *Underwood* model on speed and density data collected in two specific conditions. Also, for further adaption new models were presented for two conditions.

Data Collection Methodology

In highway speed control condition (on Tehran urban highways), the flow is assumed Homogeneous and in highway quasi-free-flow condition (on the suburban highways) at the entrance of city, the flow rate is considered Heterogeneous.
In the Heterogeneous traffic stream, all vehicles were classified into 5 classes which are defined by length of vehicle, 0 to 5.5 meters length (class 1 such as passenger cars), 5.5 to 7 meters (class 2 such as minibus and small trucks), 7 to 10 meters (class 3 such as 3-axle or 2-axle tucks), 10 to 12.5 meters (class 4 such as buses) and over 12 meters (class5 such as semi-trailer trucks).

Highway speed control condition

The data collected at 4 stations on Niayesh highway were investigated. The traffic data were acquired by radar traffic counter systems installed on speed cameras. The information includes the time mean speed and the number of vehicles in 15-minute intervals in weekdays for a three-month period. The space mean speed was calculated using *Wardrop* equation (equation 1) and the density was calculated using the fundamental traffic flow equation proposed in 1955 by Lighthill and Whitham [3].

$$\bar{U}_T = \bar{U}_S + \left(\frac{\sigma^2{}_S}{\bar{U}_S}\right) \quad (1)$$

Where

$\sigma^2{}_S$ = the variance of space mean speed, $\bar{U}_S$ = space mean speed and $\bar{U}_T$ = time mean speed.

Highway quasi-Free- Flow condition

The study on Heterogeneous flow was based on the data acquired from induction loop detectors installed in Tehran-Saveh highway. The spot speed of different classes was measured for each lane separately over 15-minute interval for a three-month period. The extracted data was labeled with the type of vehicle, lane number and weekday information. The space mean speed was calculated using the spot mean speed data by equation 1, and density was determined based on percent occupancy time of loop by equation 2 [3].

$$K = \frac{10}{L_V + L_D} (\%OCC) \quad (2)$$

Where

K = density (veh/km/lane) ,L_V = average vehicle length (m) ,L_D = average sensor length (m) (is set to 2 meters) and OCC% =$\left(\frac{\Sigma(t_i)_0}{T}\right)$.

Research Method

In this research, before conducting the analysis, some pre-processing of the data was done, to remove those data points when special events such as highway crashes or severe weather events occurred.

Due to the absence of the non-passenger vehicle in homogeneous traffic, speed and density related to this class were used as a dependent and independent variables, respectively. In heterogeneous traffic, because of the presence of other classes, passenger car speed was used as a dependent variable (S) and vehicle density for other classes were used as independent variables.

To determine the effects of all variables on the passenger car speed, regression analysis was used. It should be noted that the multiple regressions presented herein are not intended to be a universal predictive passenger car speed model. In this research, stepwise regression technique was used to confirm the results from multiple regressions. This method starts with no variables and then at each step, variables are added to the model one by one until the addition of the next variable make no significant difference.

First, the compatibility of three known models (*Greenshilds*, *Greenberg* and *Underwood*) was assessed with actual data collected in both types of traffic stream (homogenous and heterogeneous).

In continuation of this study, a new model for homogenous traffic stream and a proposed method for better fitting effectiveness with heterogonous traffic data were offered. Actually, in the proposed method three known model have been developed using new variables $\{D_1, D_2, D_3, D_4, D_5\}$.

Model Compatibility

Model Compatibility in Homogeneous Flow

The goodness-of-fit of the regression model, used for evaluating the estimation results, can be measured by the R-squared value (Table 1). After comparing the compatibility rate of three models with actual data, it can be found that the compatibility rate of *Greenshields* model (with lowest R-squared) is worst and this model shows a lower accuracy. Among models studied, *Underwood* model had the best compatibility with highest R-squared value. However, the three models could be considered as suitable models with praiseworthy compatibility results on speed and density data in homogeneous flow.

Model compatibility in heterogeneous flow

Like Homogeneous flow, the R-squared value was measured as the goodness-of-fit of the regression model. Table 2 illustrates the compatibility of three models on passenger car speed and density data in suburban highways at the entrance of Tehran for each lane (Tehran-Saveh).

Since passenger car speed(S) is affected by not only D1 but also other vehicle types, a high compatibility cannot be expected on passenger car speed and density data in heterogeneous flow. Looking at the speed-density scattered data (Fig. 1) and the low R-squared value (Table 2), the inaccuracy of these models in such flow can be concluded.

Table 1. R-squared Values for Three Models in Homogeneous Flow

Station no.	Model			Difference Between Max And Min R-square
	Greenshields	***Greenberg***	***Underwood***	
1	0.842[a]	0.889	0.892[b]	5.93%
2	0.722[a]	0.813[b]	0.781	12.6%
3	0.969[a]	0.976	0.98[b]	1.12%
4	0.836[a]	0.879	0.88[b]	5.26%

a. Minimum value of correlation b. Maximum value of correlation

Table 2. R-squared Values for Three Models in Heterogeneous Flow

Lane no.	Model		
	Greenshields	***Greenberg***	***Underwood***
3(inner lane)	$S=-1.36D_1+81.7$ $R^2=0.23$	$S=-2.7Log(D_1)+79.8$ $R^2=0.31$	$S=80.4e^{-.02D_1}$ $R^2=0.21$
2(medial lane)	$S=-.06D_1+98.8$ $R^2=0.20$	$S=-0.3Log(D_1)+99.0$ $R^2=0.29$	$S=97.3e^{-.01D_1}$ $R^2=0.18$
1(outer lane)	$S=-0.37D_1+109.4$ $R^2=0.31$	$S=-0.4Log(D_1)+108.7$ $R^2=0.34$	$S=107.7e^{-.01D_1}$ $R^2=0.38$

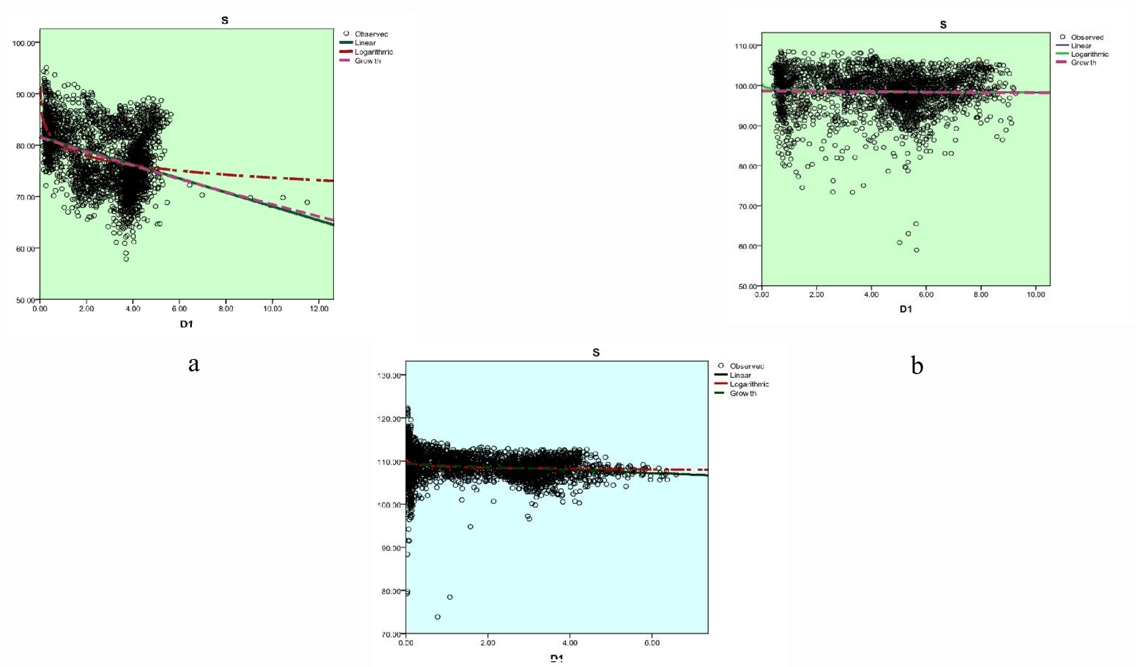

Figure 1. Speed-Density Scattered Data (a) Lane 1 (b) Lane 2 (c) Lane 3

Proposed Model

Highway speed control condition

In order to improve the prediction of the passenger car speed in highways with certain speed limit, a new model was proposed.

According to speed-density diagram (Figure 2), in the region with high density (i.e. $D>D_O$), with decreasing density up to D_O, speed increases. This trend continues until a certain speed (S_o).The S_o Value varies in different sections and depends on traffic condition situations. Studies show that due to the mental influence of enforcement rules on drivers, passenger car speed is even slower than the permissible speed limit. Therefore, after the mean speed reaches S_o, the passenger car speed decreases and reaches S_f value. In this situation, this model is a dual-regime model in which a linear model is specified for the stable conditions (1st regime) and a non linear model is specified for forced flow conditions (2nd regime) as shown in Figure 2.

Seven parameters are affecting the shape of it. In this study the proposed model will be referred as "Sarkar dual-regime model" which is presented as equation 3.

For forced flow $(D > D_0)$

$$S < S_{min} \quad S = S_{min} - (S_{min} - S_O)\left(\frac{D_O}{D}\right)^n \quad (Sarkar\ dual - regim\ model)$$

For stable flow $(D < D_0)$

$$S = S_f + \left(\frac{S_o - S_f}{D_o}\right) \times D \qquad (Sarkar\ dual - regim\ model) \qquad (3)$$

Where

S_{min} is observered minimum speed

n is a power term giving the highest R-squared value in each station

$D$is defined as passenger car density in homogenous traffic

S_{min} is observered minimum speed

$S_f$is free speed (corresponding to $D = 0$)

$D_o$is Optimal density (corresponding to maximum flow)

S_o is Optimal speed (corresponding to maximum flow)

S is Average speed

D_j is Jam density(for Greenberg model)

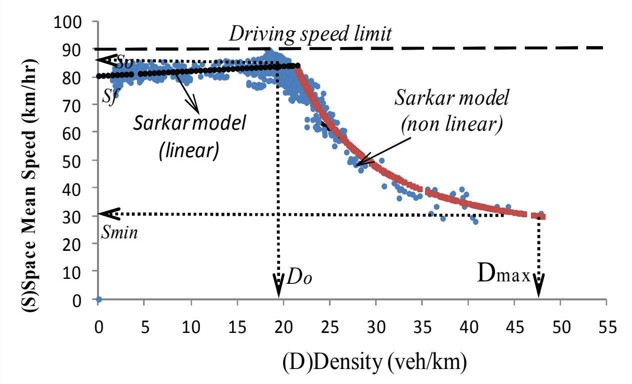

Figure 2. (a) Speed-Density diagram at the Station 4.

Highway quasi-free-flow condition

Using the step wise regression method and the new variables, linear, logarithmic and exponential models were calculated. Involving all classes as new variables and considering weekday and lane number as constant values in the model, the model coefficients are presented in equations 4,5 and 6.

Linear model:

$$S = -.389D_1 - 1.999D_2 - 20.511D_4 - 4.721D_5 + 100.788 + C_D + C_L \quad (4)$$
$$R^2 = .91; DW = 1.401; sig = 0.0000$$

Logarithmic model:

$$LnS = -.01LnD_1 - .008LnD_2 - .004LnD_3 - .003LnD_4 + 4.603 + C_D + C_L \quad (5)$$
$$R^2 = .70; DW = 1.551; sig = 0.000$$

Exponential model:

$$S = exp\{-.004D_1 - .027D_2 - .155D_4 - .045D_5 + 4.633 + C_D + C_L\} \quad (6)$$
$$R^2 = .70; DW = 1.585; sig = 0.0000$$

In above relationships, quantitative parameters are appeared as the variables with their coefficients and the qualitative parameters are presented as the constants.
the passenger car speed (S) is presented as a dependent variable and vehicle densities for the other classes are presented as independent variables $\{D_1, D_2, D_3, D_4, D_5\}$ .Also, to evaluate the effect of lane number and weekday on the passenger car speed model , a lane constant C_L and a day constant C_D was defined for each model. These values are presented in table 3 and 4.
Table 5 shows the unstandardized and standardized coefficients, t–test, statistical significance and collinearity statistics for passenger car speed.
According to this table, independent variables included in the model are statistically significant. Also, because of small variance inflation factor (VIF<10), there was no concern about the multicollinearity. This factor measures how much the variance of a regression coefficient is increased due to collinearity [17].

Table 3.Day Constant values

	Day Constant (C_D)						
	Saturday	**Sunday**	**Monday**	**Thursday**	**Wednesday**	**Tuesday**	**Friday**
Linear	-0.829						2.166
Logarithmic			-0.015	-0.009	-0.007		0.033
Exponential	0.009		-0.008				0.023

Table 4.Line Constant values

	Lane Constant (C_L)		
	Line 1 (Outer lane)	**Line 2 (medial lane)**	**Line 3 (inner lane)**
Linear	-۱۸,۶۹۷		۷,۱۱۷
Logarithmic	-0.208		۰,۰۶۹
Exponential	-0.186		0.064

Table 5. Regression coefficients for exponential model in the last two steps

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients			Collinearity Statistics	
		B	Std. Error	Beta	t	Sig.	Tolerance	VIF
8	(Constant)	4.632	.003		1502.505	.000		
	LINE1	-.186	.003	-.568	-55.717	.000	.356	2.806
	LINE3	.064	.003	.196	22.695	.000	.495	2.020
	D2	-.027	.005	-.090	-4.946	.000	.111	9.030
	Friday	.024	.003	.054	8.235	.000	.850	1.177
	D4	-.155	.022	-.063	-7.107	.000	.467	2.140
	D1	-.004	.001	-.050	5.841	.000	.499	2.005
	D5	-.045	.008	-.082	-5.287	.000	.153	6.525
	Saturday	.010	.003	.022	3.631	.000	.965	1.037
9	(Constant)	4.633	.003		1473.000	.000		
	LINE1	-.186	.003	-.569	-55.800	.000	.356	2.809
	LINE3	.064	.003	.196	22.677	.000	.495	2.020
	D2	-.027	.005	-.089	-4.889	.000	.111	9.034
	Friday	.023	.003	.051	7.632	.000	.824	1.214
	D4	-.155	.022	-.063	-7.076	.000	.467	2.140
	D1	-.004	.001	-.049	-5.678	.000	.497	2.011
	D5	-.045	.008	-.082	-5.300	.000	.153	6.525
	Saturday	.009	.003	.019	3.039	.002	.929	1.076
	Monday	-.008	.003	-.017	-2.746	.006	.936	1.068

a. Dependent Variable: S

Model Evaluation

Before utilizing the model, the model verity must be evaluated. In step wise regression method, the first model evaluation is checking the standardized residuals to be normally distributed with zero mean. The standardized residual is the residual divided by its standard deviation. The residuals distribution estimated with a normal distribution is illustrated in Fig 3. Results show mean value near to zero and variance value near to one. Thus, it can be concluded that the residuals are normally distributed.

The second model evaluation is considering the sig value which presents punctuality of the model. Sig<0.05 shows more than 95 percent punctuality. In this experiment, the sig values validate the models punctuality [17]. The third model evaluation is testing the Durbin-Watson (DW) statistic which is a measure of autocorrelation. Autocorrelation occurs when the error terms in a regression model are not independent, in other words, the value of time period in the prediction model are affecting the values of current period. An acceptable range is from 1.5 to 2.5 [17].

$$DW = \frac{\sum(e_t - e_{t-1})^2}{\sum e_t^2} \quad (7)$$

Where $e_t$the error is value of the current period and $e_{t-1}$is the error value of the previous time period.

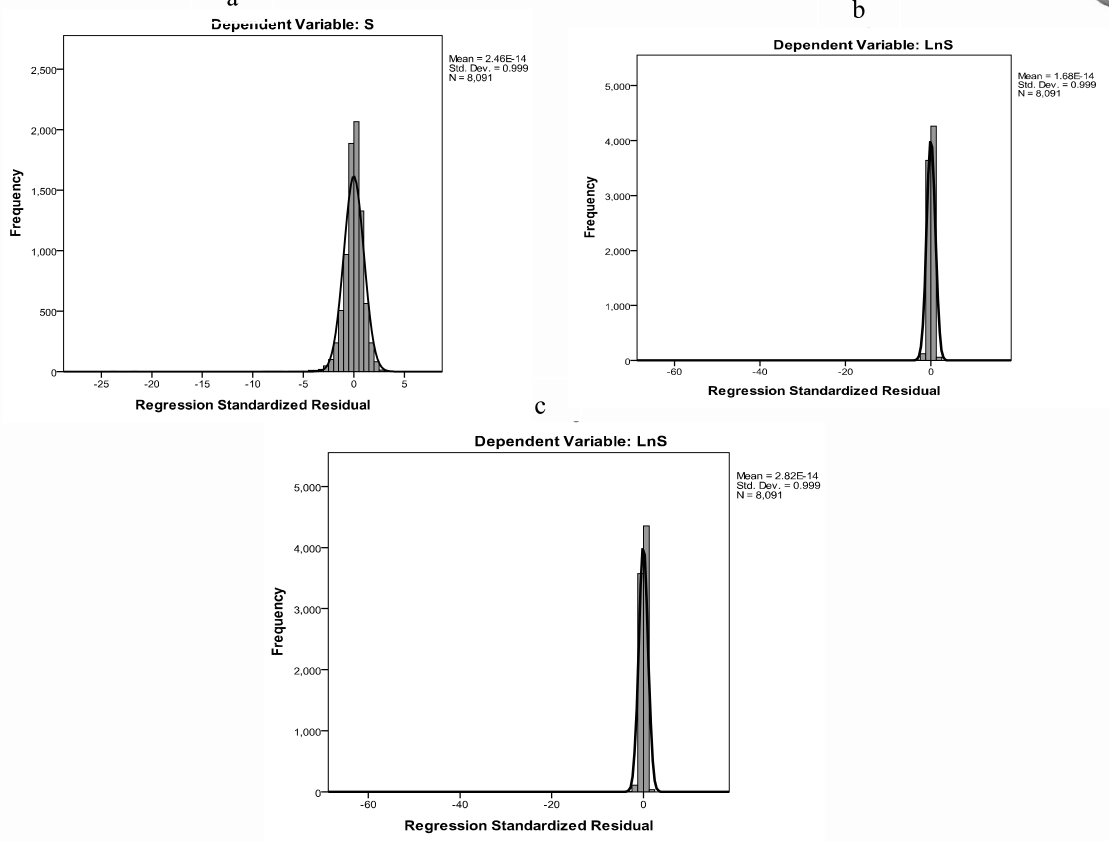

Figure 3. Model Error Distribution Estimated with a Normal Distribution in (a) Linear (b) Logarithmic (c) Exponential models.

Table 6. R-squared and "n" Values for Sarkar dual-Regime Model

Station no.	R-squared (R^2)	n Value
1	0.98	1.2
2	0.98	0.1
3	0.985	1.6
4	0.935	2.6

Results

Highway speed control condition

According to Table 6, R-squared values of "Sarkar dual-regime model" are high at all stations. Due to disability of drivers to keep their speed near the speed limit (90km/hr) and speed corresponding to maximum flow rate, The free flow speed (F.F.S) varies between S_o and S_f (S_f< F.F.S < S_o). Consequently, in this region, a scatter of data is so wide that the models were incompatible with the speed and density data. Among the logarithmic, exponential and linear models the linear model had a suitable compatibility with the acquired data in stable flow region. As an example, these trends were illustrated in Figure 2.

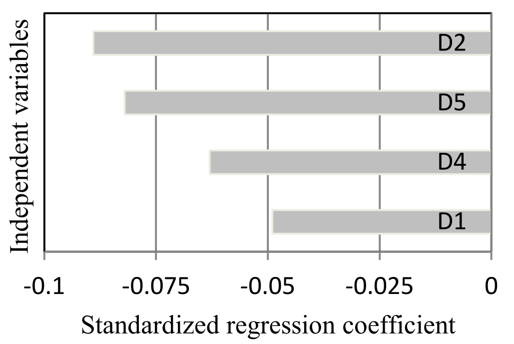

Figure 4. ranking the importance of the independent variables exponential model

Quasi-free-flow condition

Standardized coefficients convert all the variables into standard deviation units. This conversion is usually done to answer the question of which of the independent variables have greater effects on the dependent
variable in a multiple regression analysis [18].

It can be seen from the Table 5 and 7 that all independent variables did not include in the passenger car speed model. This could be because some of them are not statistically significant. As an example, variable D_3 was omitted from the exponential model and variable D_5 from the logarithmic model. The only common variables in both models were included: D_1, D_2 and D_4. Figure 4 shows the variables with the highest and lowest influence on the passenger car speed(S).The negative sign indicates that the higher the vehicle density, the lower the passenger car speed. The beta value for Variable D_2 is maximum in both models. In other words, mini bus and small truck density could be identified as the most effective factor on the passenger car speed. The effect of different variables on the variable S depends on the model type.

The goodness-of-fit of three regression models were measured by the R-squared, DW and Sig value. Analysis shows that with addition of the effects of other vehicles along with weekdays and lane numbers the logarithmic and exponential model show further improvement of compatibility to the data (considering the DW test). For these models, DW is greater than 1.5 that is acceptable. Also, the new changes have caused the correlation to be better up to three times (R2=0.70).Table 6 and 8 show that 70% of variance for “S” is explained by the independent variable. But due to the greater DW for exponential model (1.585), this model is more appropriate.

Table 7. Summary of Statistic Parameters for Various models

	Linear	***Logarithmic***	***Exponential (Growth)***
R-squared	0.91	0.697	0.701
DW	1.401	1.551	1.585
Sig	0.000	0.000	0.000

Conclusion

This study was based on the speed and density data acquired from an urban and a suburban highway which presented highway speed control condition and highway quasi-free-flow condition, correspondingly. The model compatibility of *Greenberg*, *Greenshields* and *Underwood* was studied on speed and density data and for further compatibility a developed model was proposed at each flow regime. The results of this research showed that:

1-Speed limitation can affect traffic flow by changing the values of some model parameters such as free speed.

2- In highway speed control condition, the new dual-regime model had higher compatibility on all speed and density data. Using new model the passenger car mean speed could be estimated more accurately.

3- In highway quasi-free-flow condition, by considering the effect of new parameters ,*Greenberg* and *Underwood's* models could result in an accurate prediction of passenger car speed.

4-In quasi-free- flow regimes, the passenger car speed (S) is more influenced by mini-bus and small trucks.

Acknowledgment

The writers would like to thank Tehran traffic control company (TTCC) for providing traffic data.

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